Modal Information Logics: Axiomatizations and Decidability

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Based on MSc thesis, supervised by Johan van Benthem and Nick Bezhanishvili
February 24, 2023

Universiteit van Amsterdam

Plan for the talk

- Big picture, few details (so please let me know if you'd like elaboration)
- Outline of the talk
    1. Introducing the logics
    2. Stating the problems
    3. Outlining the strategy
    4. Solving the problems using the strategy
- Overarching theme: a study of modal information logics

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Defining (the basic) modal information logics (MILs)

Definition (language and semantics)

The language is given by

$$\varphi ::= \bot \mid p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle \sup \rangle \varphi\psi,$$

and the semantics of '$\langle \sup \rangle$' is:

$$w \Vdash \langle \sup \rangle \varphi\psi \quad \text{iff} \quad \exists u, v (u \Vdash \varphi;\ v \Vdash \psi;\ w = \sup\{u, v\})$$

Example

Definition (frames and logics)

Three classes of frames $(W, \leq)$, namely those where

(Pre) $(W, \leq)$ is a preorder (refl., tr.);

(Pos) $(W, \leq)$ is a poset (anti-sym. preorder); and

(Sem) $(W, \leq)$ is a join-semilattice (poset w. all bin. joins)

Resulting in the logics MIL_{Pre}, MIL_{Pos}, MIL_{Sem}, respectively.

Appetizer: Let's show that $MIL_{Pre} \subseteq MIL_{Pos} \subsetneq MIL_{Sem}$. **see blackboard**

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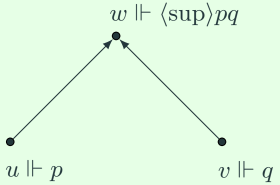

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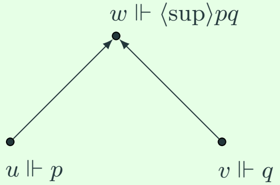

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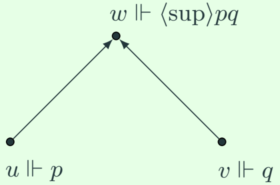

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Why MILs?

- Connect with other logics (e.g., truthmaker logics).
- Introduced to model a theory of information (by van Benthem (1996)).
- Modestly extend $\mathbf{S4}$ [MIL_{Pre}, MIL_{Pos}]. **see blackboard**

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Proposition

MILs lack the finite model property (FMP) w.r.t. their classes of definition. **see blackboard**

How we solve (A), and then (D) using (A):

(1) We axiomatize MIL_{Pre} (and deduce $MIL_{Pre} = MIL_{Pos}$).

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MIL$_{Pre}$ is (sound and complete w.r.t.) the least normal modal logic with axioms:

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Proof idea

Soundness *see blackboard* ✓

For completeness, let $\Gamma \supseteq \Gamma_0$ be an MCS extending some consistent Γ_0. We construct a satisfying model using the step-by-step method (but first, why step-by-step? *see blackboard*).

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Completeness of MIL_{Pre} (cont.)

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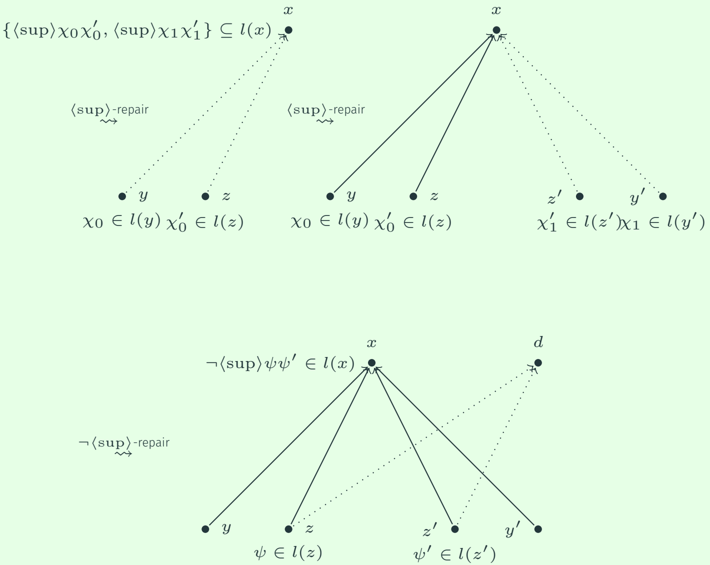

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(2) and (3): 'decidability via completeness'

(2) Find another class $\mathcal{C}$ for which $\mathrm{Log}(\mathcal{C}) = \mathit{MIL}_{Pre}$:
   (i) Nothing in the ax. of MIL_{Pre} necessitating '$\langle \sup \rangle$' to be interpreted using a supremum relation.
   (ii) Canon. re-interpretation:

$$\mathcal{C} := \{(W, C) \mid (W, C) \Vdash (Re.) \wedge (Co.) \wedge (4) \wedge (Dk.)\},$$

   where $C \subseteq W^3$ is an arbitrary relation.
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Thus, we have solved both (A) and (D).

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Can we generalize these techniques?

MILs with informational implication '\'

(Natural) extensions of MIL_{Pre} and MIL_{Pos} [and $\mathbf{S4}$] are obtained by adding an informational implication '\'.

Definition

The language is given by adding '\' with semantics:

$$v \Vdash \varphi \backslash \psi \qquad \text{iff} \qquad \forall u, w([u \Vdash \varphi, w = \sup\{u, v\}] \Rightarrow w \Vdash \psi)$$

We denote the resulting logics as $MIL_{\backslash\text{-}Pre}, MIL_{\backslash\text{-}Pos}$, respectively.

Note that '$\langle \sup \rangle$' and '\' are "inverses"; and 'F' is expressible: we extend temporal **S4**. **see blackboard**

The problems now become

(A\) axiomatizing $MIL_{\backslash\text{-}Pre}$ and $MIL_{\backslash\text{-}Pos}$; and
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The same (1)-(2)-(3) structure is used as before, but now we

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Selected points from proof of (A\), (D\) through (1'), (2'), (3')

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$\mathrm{Log}_\backslash(\mathcal{C})$ is (sound and complete w.r.t.) the least set of $\mathcal{L}_{\backslash\text{-}M}$-formulas that (i) is closed under the axioms and rules for MIL_{Pre}; (ii) contains the K-axioms for \; (iii) contains the axioms

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About the proof

Soundness: routine; completeness: standard.

Lambek Calculus of suprema on preorders/posets

This logic (which $=$ $MIL_{\backslash\text{-}Pre}$ $=$ $MIL_{\backslash\text{-}Pos}$) $=$ **NL-CL** $+ \{(Re.), (4), (Co.), (Dk.)\}$, where **NL-CL** is the Lambek Calculus extended with CL from, e.g., Buszkowski (2021).

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This concludes and summarizes our study of MILs on preorders and posets.

How about join-semilattices (i.e., MIL_{Sem})?

Three ways to completeness (some intuitions for our proof):

Henkin (e.g., $\mathbf{K}$)

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'Indeterministic step-by-step' (MIL_{Sem})

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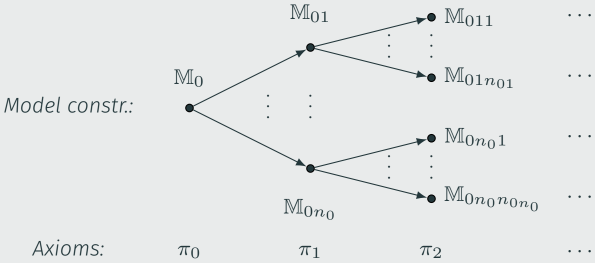

Conclusion and future work

What we have done:

- Surveyed the landscape of MILs on preorders and posets.
- Made crossings with the Lambek Calculus and truthmaker logics.[1]
- Axiomatized MIL_{Sem}.

What (might) come next:

- Proving (un)decidability of MIL_{Sem} and solving the ancillary problems of fin. ax. and the FMP w.r.t. $\mathcal{C}_{Sem}$.
- Further exploring how MILs connect to other logics.

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Thank you!

References I

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On '$\backslash$' and '$\langle\mathrm{sup}\rangle$'

Example

Note how '$\langle\mathrm{sup}\rangle$' and '$\backslash$' are 'inverses':

$$\langle\mathrm{sup}\rangle p(p\backslash q) \to q$$

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